Anfängerpraktikum: 3D Latex-Ausgabe für Geogebra

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Was ist Geogebra?

- Interaktive Geometrie-Software
- Außerdem Algebra-, Statistik-, Analysis-Software
- Web-Anwendung
- Desktop-Client verfügbar
- Teile des Programms sind unter GNU General Public License als Freie Software verfügbar
- Allgemein kostenlos für nicht-kommerzielle Nuztung

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2D-Graphing

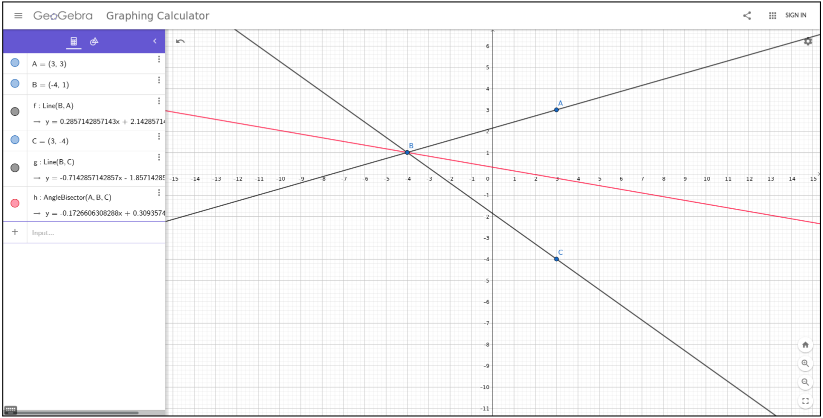

Figure 1: 2D Graphing Example

3D-Graphing

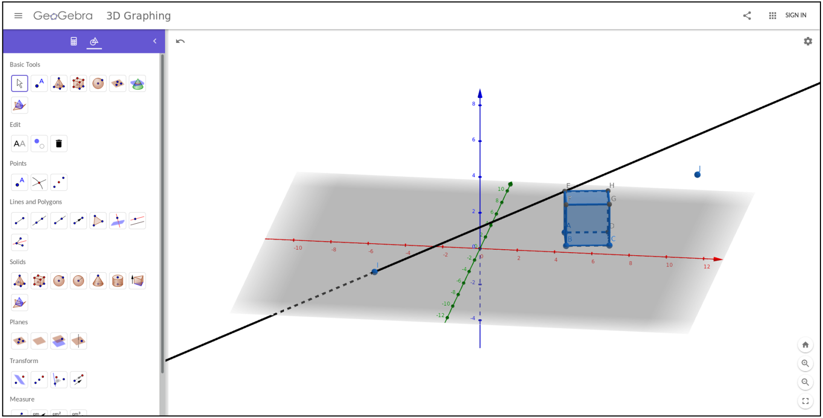

Figure 2: 3D Graphing Example

Geogebra-Export

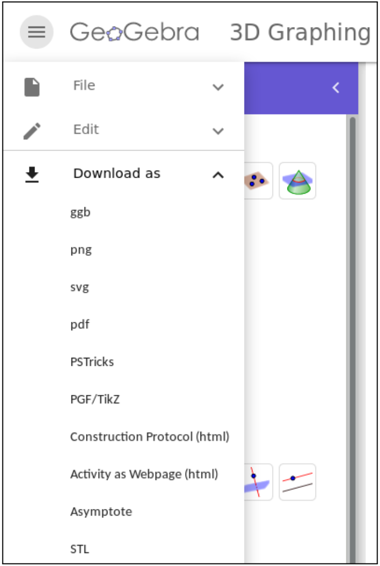

Figure 3: Export Funktion

PNG-Export

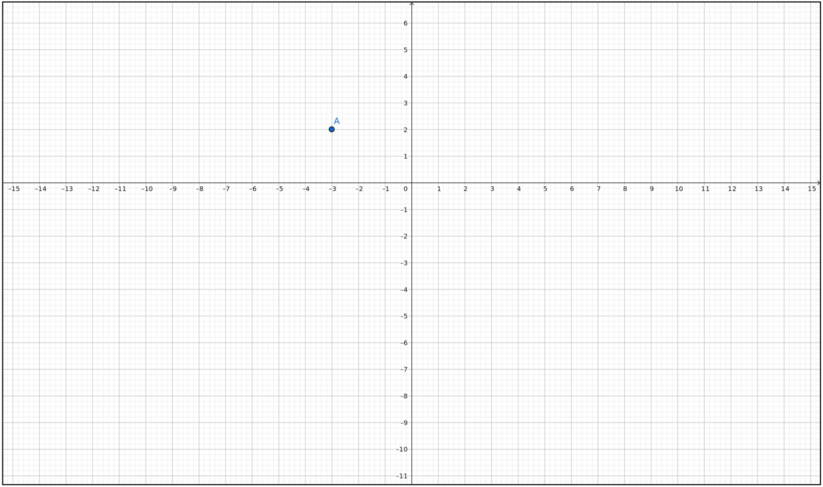

Figure 4: PNG - Das einfachste Beispiel: Ein Punkt

LaTeX -Beispiel

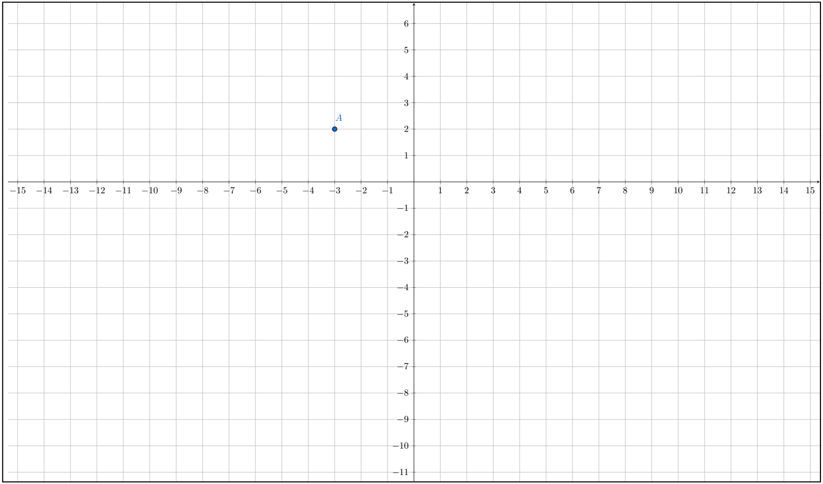

Figure 5: LaTeX - Das einfachste Beispiel: Ein Punkt

LaTeX -Code

```
\documentclass[10pt]{article}
\usepackage{pgf,tikz,pgfplots}
\pgfplotsset{compat=1.15}
\usepackage{mathrsfs}
\usetikzlibrary{arrows}
\pagestyle{empty}
\begin{document}
\definecolor{rvwvcq}{rgb}{0.08235294117647059,0.396078431372549,0.7529411764705882}
\begin{tikzpicture}[line cap=round,line join=round,>=triangle 45,x=1cm,y=1cm]
\begin{axis}[
x=1cm,y=1cm,
axis lines=middle,
ymajorgrids=true,
xmajorgrids=true,
xmin=-15.360000000000003,
xmax=15.360000000000003,
ymin=-11.34,
ymax=6.78,
xtick={-15,-14,...,15},
ytick={-11,-10,...,6},]
\clip(-15.36,-11.34) rectangle (15.36,6.78);
\begin{scriptsize}
\draw [fill=rvwvcq] (-3,2) circle (2.5pt);
\draw[color=rvwvcq] (-2.84,2.43) node {$A$};
\end{scriptsize}
\end{axis}
\end{tikzpicture}
\end{document}
```

Figure 6: LaTeX Farben

LaTeX -Code

```
\documentclass[10pt]{article}
\usepackage{pgf,tikz,pgfplots}
\pgfplotsset{compat=1.15}
\usepackage{mathrsfs}
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\pagestyle{empty}
\begin{document}
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\begin{tikzpicture}[line cap=round,line join=round,>=triangle 45,x=1cm,y=1cm]
\begin{axis}[
x=1cm,y=1cm,
axis lines=middle,
ymajorgrids=true,
xmajorgrids=true,
xmin=-15.360000000000003,
xmax=15.360000000000003,
ymin=-11.34,
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ytick={-11,-10,...,6},]
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\draw[color=rvwvcq] (-2.84,2.43) node {$A$};
\end{scriptsize}
\end{axis}
\end{tikzpicture}
\end{document}
```

Figure 7: LaTeX - Achsen, clipping field

LaTeX -Code

```
\documentclass[10pt]{article}
\usepackage{pgf,tikz,pgfplots}
\pgfplotsset{compat=1.15}
\usepackage{mathrsfs}
\usetikzlibrary{arrows}
\pagestyle{empty}
\begin{document}
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\begin{tikzpicture}[line cap=round,line join=round,>=triangle 45,x=1cm,y=1cm]
\begin{axis}[
x=1cm,y=1cm,
axis lines=middle,
ymajorgrids=true,
xmajorgrids=true,
xmin=-15.360000000000003,
xmax=15.360000000000003,
ymin=-11.34,
ymax=6.78,
xtick={-15,-14,...,15},
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\clip(-15.36,-11.34) rectangle (15.36,6.78);
\begin{scriptsize}
\draw [fill=rvwvcq] (-3,2) circle (2.5pt);
\draw[color=rvwvcq] (-2.84,2.43) node {$A$};
\end{scriptsize}
\end{axis}
\end{tikzpicture}
\end{document}
```

Figure 8: LaTeX - Punkte zeichnen

.ggb-Archive

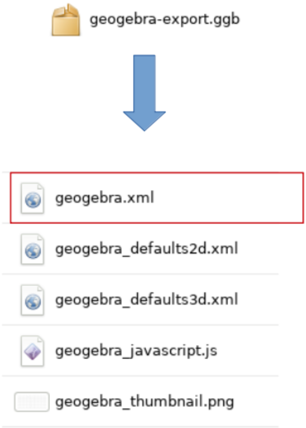


Figure 9: .ggb-Archive

XML

- Extensible Markup Language kurz: XML
- Auszeichnungssprache (vgl. HTML)
- Darstellung hierarchisch strukturierter Daten in einer Textdatei
- lesbar für **Mensch** und **Maschine**

XML-File

```
-<construction title="" author="" date="">
  -<element type="point" label="A">
      <show object="true" label="true"/>
      <objColor r="21" g="101" b="192" alpha="0"/>
      <layer val="0"/>
      <labelMode val="0"/>
      <animation step="1" speed="1" type="1" playing="false"/>
      <coords x="-3" y="2" z="1"/>
      <pointSize val="5"/>
      <pointStyle val="0"/>
   </element>
 </construction>
```

Figure 10: Punkt in XML-File

XML-File


```
-<euclidianView>
  <viewNumber viewNo="1"/>
  <size width="1536" height="906"/>
  <coordSystem xZero="768" yZero="339" scale="49.99999999999999" yscale="50"/>
  <evSettings axes="true" grid="true" gridIsBold="false" pointCapturing="3" rightAngleStyle="1"
  <bgColor r="255" g="255" b="255"/>
  <axesColor r="0" g="0" b="0"/>
  <gridColor r="192" g="192" b="192"/>
  <lineStyle axes="1" grid="0"/>
  <axis id="0" show="true" label="" unitLabel="" tickStyle="1" showNumbers="true"/>
  <axis id="1" show="true" label="" unitLabel="" tickStyle="1" showNumbers="true"/>
</euclidianView>
```


Figure 11: Clipping Field

Unterschiede XML-Files

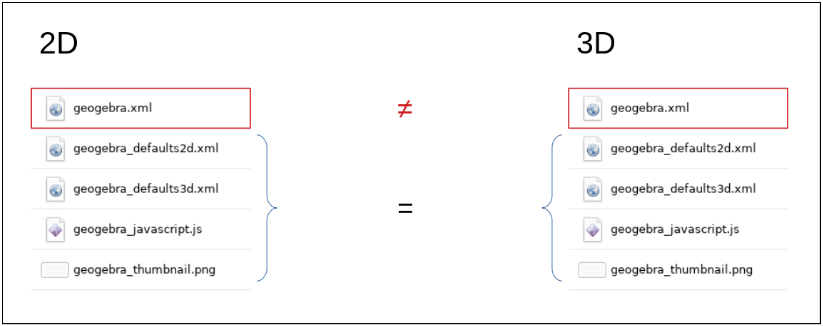


Figure 12: Vergleich XML-Files

Unterschiede XML-Files


```
-<euclidianView3D>
    <coordSystem xZero="0" yZero="0" zZero="-1.5" scale="50" xAngle="20" zAngle="-60"/>
    <evSettings axes="true" grid="false" gridIsBold="false" pointCapturing="3" rightAngleStyle="1" gridType="3"/>
    <axis id="0" show="true" label="" unitLabel="" tickStyle="1" showNumbers="true"/>
    <axis id="1" show="true" label="" unitLabel="" tickStyle="1" showNumbers="true"/>
    <axis id="2" show="true" label="" unitLabel="" tickStyle="1" showNumbers="true"/>
    <plate show="true"/>
    <bgColor r="255" g="255" b="255"/>
    <clipping use="false" show="false" size="1"/>
    <projection type="0"/>
  </euclidianView3D>
-<construction title="" author="" date="">
    <expression label="A" exp="(4, -3, 2)" type="point"/>
  -<element type="point3d" label="A">
      <show object="true" label="true" ev="4"/>
      <objColor r="21" g="101" b="192" alpha="0"/>
      <layer val="0"/>
      <labelMode val="0"/>
      <animation step="1" speed="1" type="0" playing="false"/>
      <coords x="4" y="-3" z="2" w="1"/>
      <pointSize val="5"/>
    </element>
  </construction>
</geogebra>
```


Figure 13: EuclidianView3D

Unterschied 2D-3D

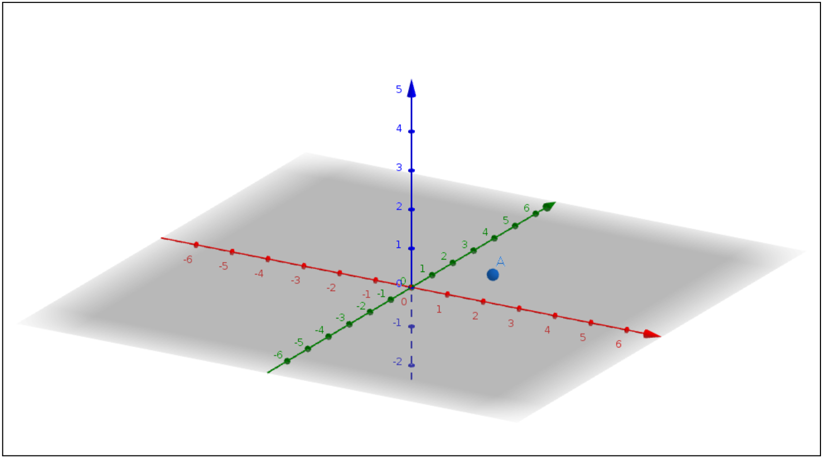


Figure 14: Ein Punkt in 3D, $A = (4, -3, 2)$

Die Diagonale

$$(\text{Länge Diagonale})^2 = (\text{Länge X-Achse})^2 + (\text{Länge Y-Achse})^2$$

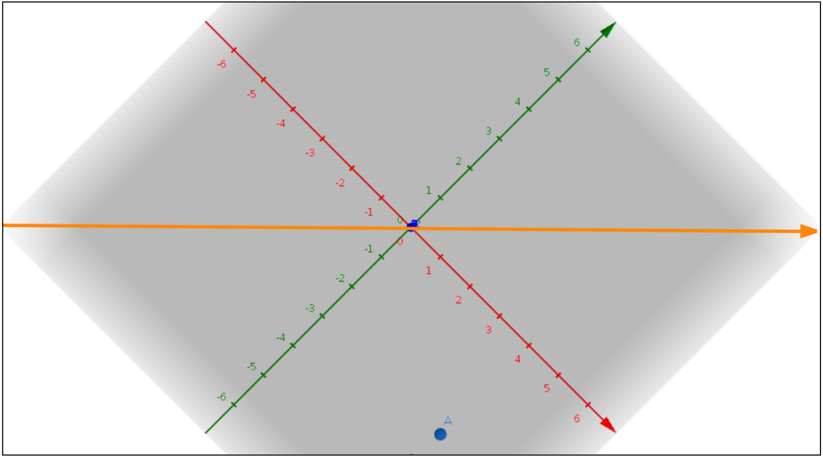
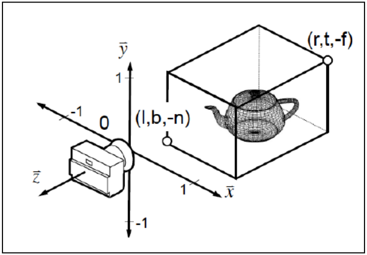
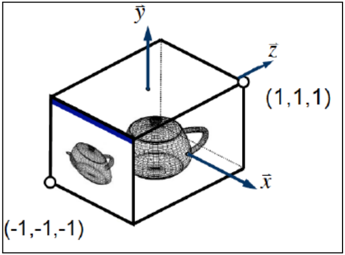


Figure 15: Die Diagonale der x-/y-Ebene

Orthographische Projektion





$$P = \begin{pmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Projektionsmatrix Beispiel

- Selbe Displaygröße bei allen Beispielen
- Width: 1366, Sidebar: 380
- Height: 546, Scale: 50 (default)

$$w := \frac{Window.width - sidebar}{scale} = 19.72$$

$$h := \frac{Window.height}{scale} = 10.92$$

Projektionsmatrix Beispiel

$$w = 19.72, h = 10.92$$

$$l = -\frac{w}{2}, \qquad n = 0, \qquad t = \frac{h}{2}$$
$$r = \frac{w}{2}, \qquad f = -w, \qquad b = -\frac{h}{2}$$

$$P = \begin{pmatrix} \frac{2}{19.72} & 0 & 0 & 0 \\ 0 & \frac{2}{10.92} & 0 & 0 \\ 0 & 0 & \frac{2}{19.72} & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Keine Drehung, keine Verschiebung

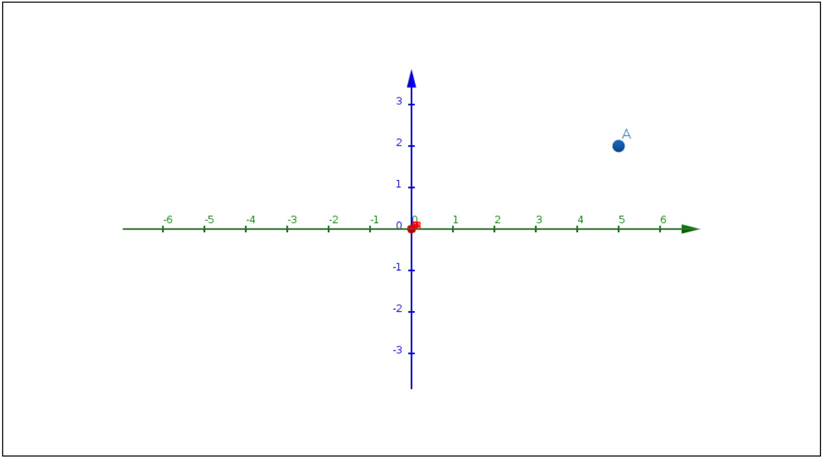


Figure 17: Geogebra-PNG,
$xZero = yZero = zZero = 0, xAngle = zAngle = 0, A = (3, 5, 2)$

Keine Drehung, keine Verschiebung

$$A = (3, 5, 2)^T, C_p = (9.86, 0, 0)^T$$

$$R_c = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, R_c^{-1} = R_c^T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Punkt in Kamera-Koordinaten:

$$A' = R_c^{-1}(A - C_p) = \begin{pmatrix} 5 \\ 2 \\ -6.86 \end{pmatrix}$$

Projektion in den Einheitswürfel:

$$E = PA_h = \begin{pmatrix} 0.507 \\ 0.366 \\ -0.696 \\ 1 \end{pmatrix}$$

Keine Drehung, Verschiebung

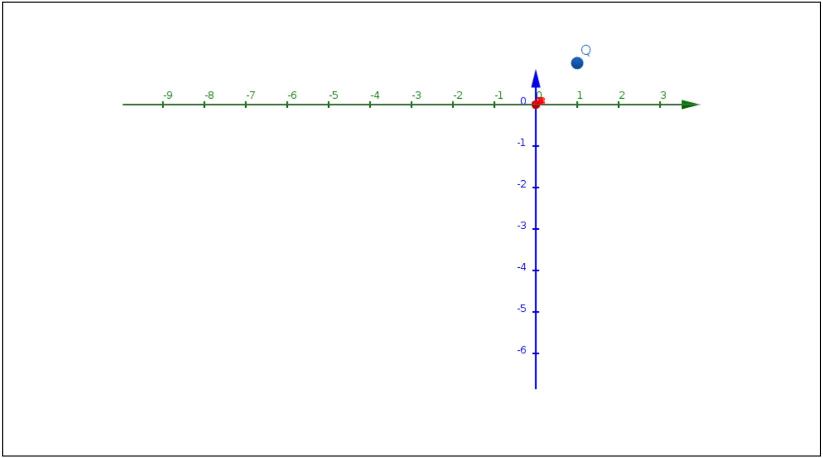


Figure 18: Geogebra-PNG,
$xZero = yZero = zZero = 3, xAngle = zAngle = 0, A = (1, 1, 1)$

Keine Drehung, Verschiebung

$$A = (1,1,1)^T, Z = (3,3,3)^T, C = (9.86,0,0)^T,$$
$$C_p = C - Z = (6.86,-3,-3)^T$$
$$R_c = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, R_c^{-1} = R_c^T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Punkt in Kamera-Koordinaten:

$$A' = R_c^{-1}(A - C_p) = \begin{pmatrix} 4 \\ 4 \\ -5.86 \end{pmatrix}$$

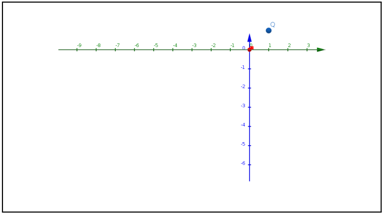
Projektion in den Einheitswürfel:

$$E = PA_h = \begin{pmatrix} 0.406 \\ 0.733 \\ -0.594 \\ 1 \end{pmatrix}$$

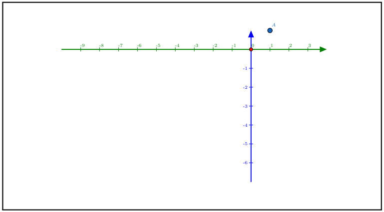
Unser LaTeX-Output

$$Y_{Anfang} = \begin{pmatrix} -7 \\ 3 \\ -6.86 \end{pmatrix}, Y_{Ende} = \begin{pmatrix} 7 \\ 3 \\ -6.86 \end{pmatrix}$$

$$Z_{Anfang} = \begin{pmatrix} 3 \\ -4 \\ -6.86 \end{pmatrix}, Z_{Ende} = \begin{pmatrix} 3 \\ 4 \\ -6.86 \end{pmatrix}$$



(a) PNG-Output



(b) Unsere LaTeX-Zeichnung

Unser LATEX-Output

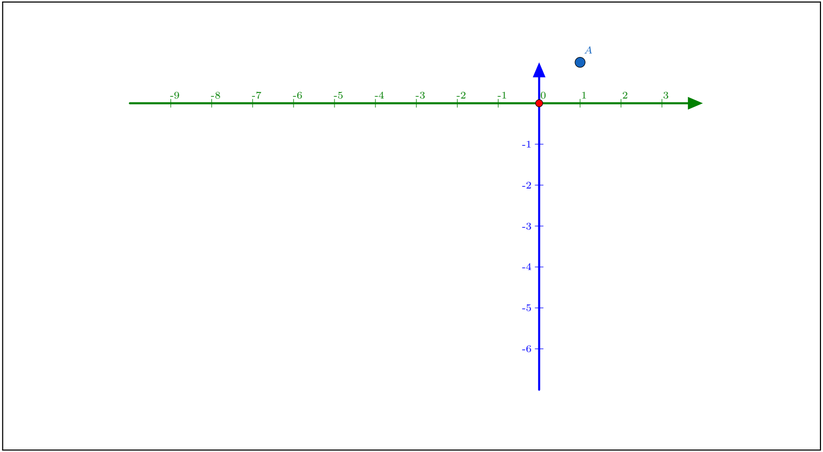


Figure 20: Unsere LATEX-Zeichnung

Unser LaTeX-Output

```
%...
\begin{tikzpicture}[line cap=round,line join=round,>=triangle 45,x=1cm,y=1cm]

\clip(-9.86,-5.46)rectangle(9.86,5.46);

\begin{scriptsize}
\draw [fill=rvwvcq] (4,4) circle (3.5pt);
\draw[color=rvwvcq] (4.2,4.3) node {$A$};

\draw [->,line width=0.5 mm,blue] (3,-4) -- (3,4);
\draw [->,line width=0.5 mm,color=ourgreen] (-7,3) -- (7,3);
\draw[fill=red](3,3) circle (2.5pt);

\foreach \x in {-9,-8,...,3}
        {
                \draw[color=ourgreen](\x+3,2.9) -- (\x+3,3.1);
                \node[color=ourgreen] at (\x+3.1,3.2) {\x};
        }
\foreach \y in {-6,-5,...,-1}
{
        \draw[blue](2.9,\y+3) -- (3.1,\y+3);
        \node[blue] at (2.7,\y+3){\y};
}

\end{scriptsize}
\end{tikzpicture}
\end{document}
```

Figure 21: Unsere LaTeX-Zeichnung

Elementare Drehmatrizen

$$R_x(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{pmatrix}$$

$$R_y(\alpha) = \begin{pmatrix} \cos(\alpha) & 0 & \sin(\alpha) \\ 0 & 1 & 0 \\ -\sin(\alpha) & 0 & \cos(\alpha) \end{pmatrix}$$

$$R_z(\alpha) = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Drehung, Keine Verschiebung

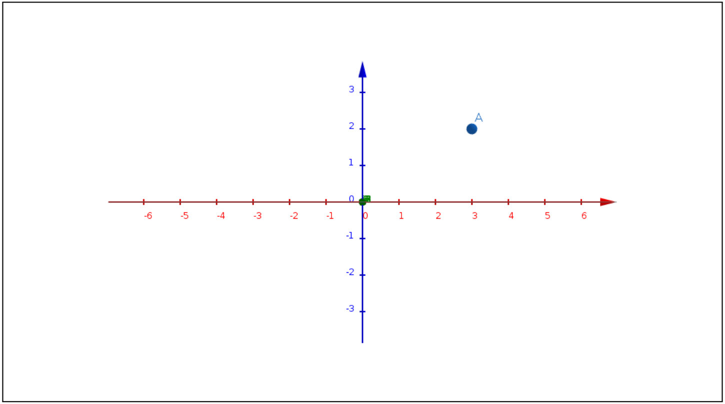


Figure 22: Unsere LaTeX-Zeichnung
$xZero = yZero = zZero = 0, xAngle = 0, zAngle = -90, A = (3, 5, 2)$

Drehung, Keine Verschiebung

$$A = (3, 5, 2)^T, C = (9.86, 0, 0)^T = C_p$$

$$R^{-1} = R = R_z(-90^\circ)R_c = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Punkt in Kamera-Koordinaten:

$$A' = R^{-1}(A - {\color{red}R_c} C_p) = \begin{pmatrix} 3 \\ 2 \\ -4.86 \end{pmatrix}$$

Projektion in den Einheitswürfel:

$$E = PA_h = \begin{pmatrix} 0.304 \\ 0.366 \\ -0.493 \\ 1 \end{pmatrix}$$

Problem: Zwei Rotationen

Zur Erinnerung:

$$A^{'} = R^{-1}(A - C_p)$$

Drehung um x-Achse (Winkel α) und z-Achse (Winkel β):

$$A^{'} = (R_z(\alpha) * R_x(\beta) * R_c)^{-1}(A - C_p)$$

Fazit

- Reverse-Engineering ist eine Erfahrung!

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- Das Zeichnen in LaTeX mit Tikz

Fazit

- Reverse-Engineering ist eine Erfahrung!
- Das Zeichnen in LaTeX mit Tikz
- Wie könnte es weiter gehen?